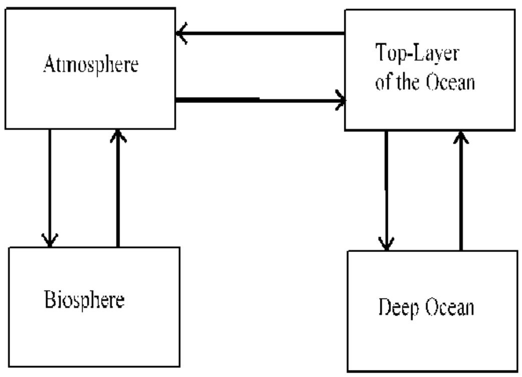
World CO2 Model Simulation with Stochastic ODEs

Intro to *Global Carbon-Dioxide (CO2) Model with Source Emission Uncertainties:*

Lumped models are also used to model transactions between different lumped sub-systems. As an example, consider the following diagram which shows the global carbon-dioxide (CO2) cycle on earth where earth is divided into four major compartments, namely, atmosphere, biosphere, the top-layer of the ocean and the deep ocean.



The arrows indicate that there is mass exchange in that direction and the mass exchange between two boxes is given as a rate coefficient*current mass. For example, net change in storage in the atmosphere is given as (we ignore boxes 3 and 4, namely, Top ocean layer and deep ocean, respectively).

$$\frac{dx_1}{\mathrm{dt}} = -k_{12}x_1 + k_{21}x_2 + s \qquad (1)$$

Similary, for the biosphere, the equation of conservation is

$$\frac{dx_2}{\mathrm{dt}} = k_{12}x_1 - k_{21}x_2 \qquad (2)$$

where kij's are rate coefficients for exchange taking place between box i to box j. Carefully note the signs which correspond to the arrows in the figure, where X = [x1, x2] are the CO2 stored in atmosphere and biosphere, respectively. The variable s in the first equation (1) includes the independence source of co2 corresponding to the emissions from human civilization.

Example: Future Predictions of CO2 Levels on Earth with uncertainty in emissions: Given that before industrialization (about year 1890), the CO2 levels as = [51 62.2] - giga tons and the rate coefficients as k12 = 1/33, k21 = 1/40, and that the source term is given as [.05*t 0] for the first 140 years simulate CO2 levels upto year 2030. The following figure shows results corresponding to the atmosphere. The scaling is in the same order as ppm co2 values in the atmosphere, approximately. So for year 2020, a little less than

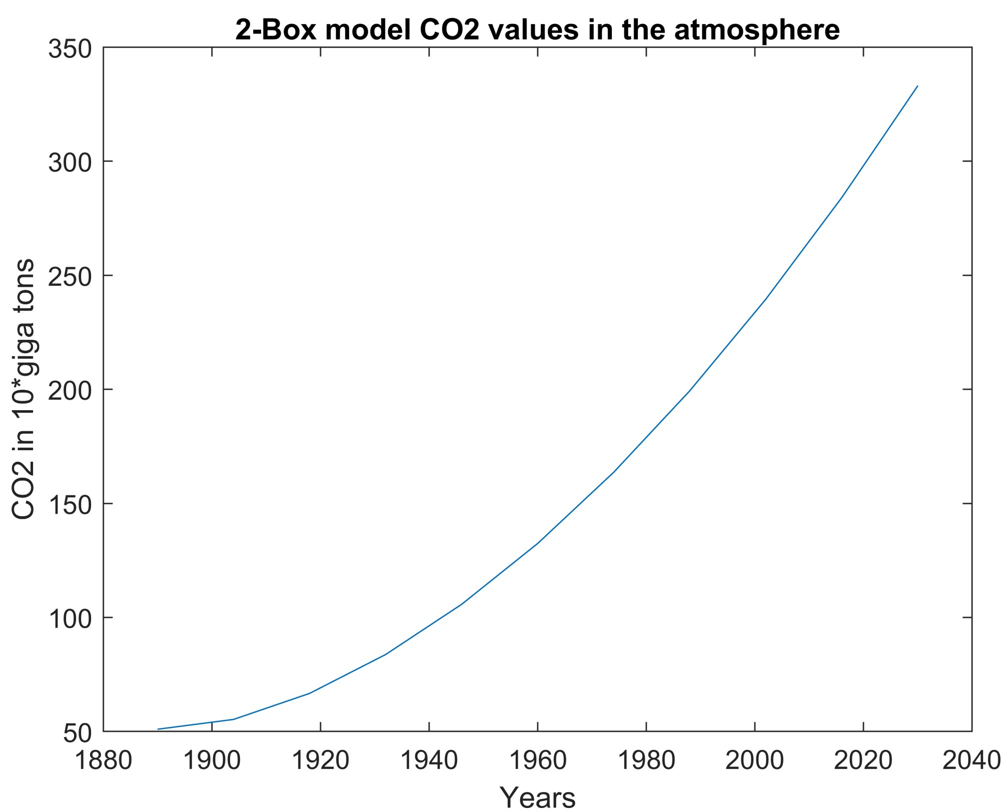
300-400 (10^10 gigatons) which is less than what we saw in the four box lumped model (where it was about 375)!

```
%main program to run the Global CO2
%model

% t0 = 1890;
% tf = 1990;
Years = [1890 2030];
x0 = [51 62.2]'; %CO2 in 10*giga tons

[t,x] = ode23('co2func2box',Years,x0);
figure
plot(t,x(:,1))


title('2-Box model CO2 values in the atmosphere')
xlabel('Years')
ylabel('CO2 in 10*giga tons')
```



However, let us now consider uncertainty in the source (emissions) only (there can be uncertainties also in the initial conditions as well as in parameters, with the latter case being a harder problem which we will discuss in another exercise).

Assuming, the uncertainty to be a white noise process N_t with intensity σ to s.

$$\frac{dx_1}{\mathrm{dt}} = -k_{12}x_1 + k_{21}x_2 + s + \sigma N_t \qquad (3)$$

However, in order to numerically integrate the above, we need *Ito stochastic calculus* and is explained elesewhere. We will only provide the numerical results here

$$dx_1 = -k_{12}x_1\mathrm{dt} + k_{21}x_2\mathrm{dt} + \mathrm{sdt} + \sigma \mathrm{dW}_t \qquad (4)$$

where dW_t is defined as

$$\mathrm{dW}_t = N_t\mathrm{dt} \qquad (5)$$

and equation has a nice numerical interpretation of

$$\mathrm{dW}_t = N_t\mathrm{dt} = N(0,1)\sqrt{\mathrm{dt}} \qquad (6)$$

which can now be used to find numerical stochastic solutions of equations (1) and (2).

Exercises:

1. Try the slider for different sigma values (intensity of uncertainty).
2. Try different number of samples from 20 to 50
3. Is the mean of the solution the same as the deterministic solution?

```
%main program to run the Global CO2
%model

% t0 = 1890;
% tf = 1990;
Years = [1890 2030];
x0 = [51 62.2]'; %CO2 in 10*giga tons
%ssig=5;
ssig=4;
Tsteps=100; %Keep this large enough so that the first-order scheme accuracy is
             % as good as we get in ode23()
figure
hold
```

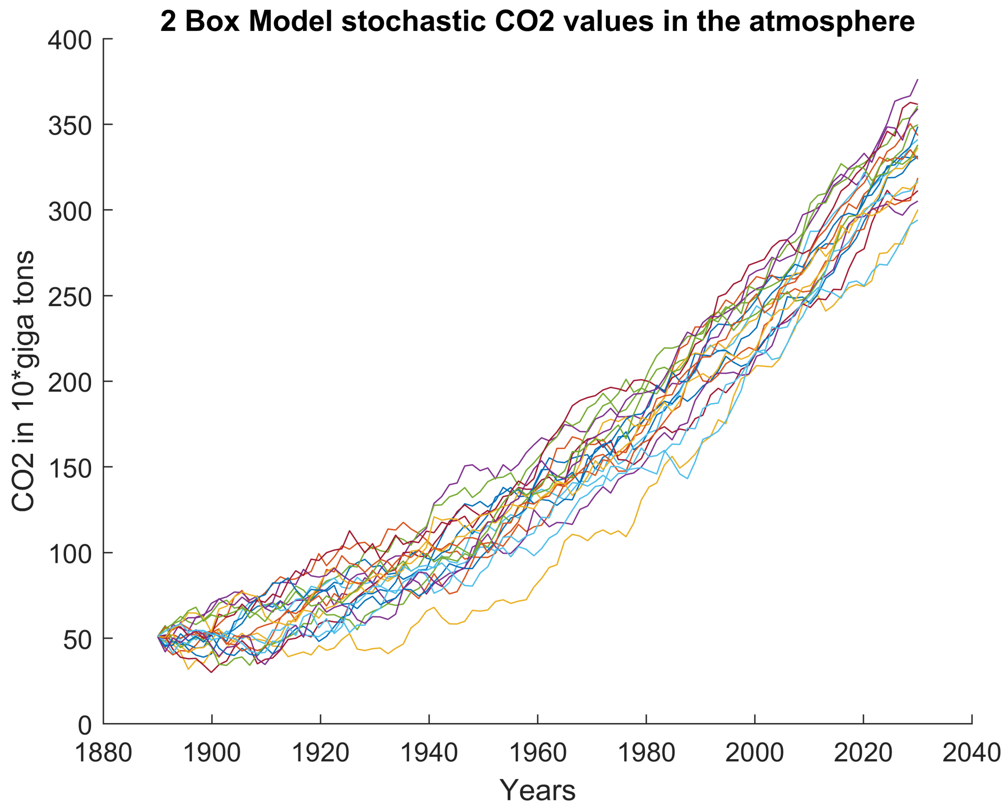
```
Current plot held
```

```
for sims=1:20
[t,x]= co2func2boxSDE(Years,x0,ssig,Tsteps);

t=1890+t;
plot(t,x(:,1))
end


title('2 Box Model stochastic CO2 values in the atmosphere')
```

```
xlabel('Years')
ylabel('CO2 in 10*giga tons')
```



References:

Ponnambalam, K., A.W. Heemink, nd S.G. Fletcher, and P. E. Kloeden. Models for Water and Environmental Systems Analysis and Design: An Interactive Webbook, University of Waterloo, Waterloo, Canada, 2010. *A chapter on Ito stochastic calculus can be found here.*

(see http://epoch.uwaterloo.ca:8008/software/)