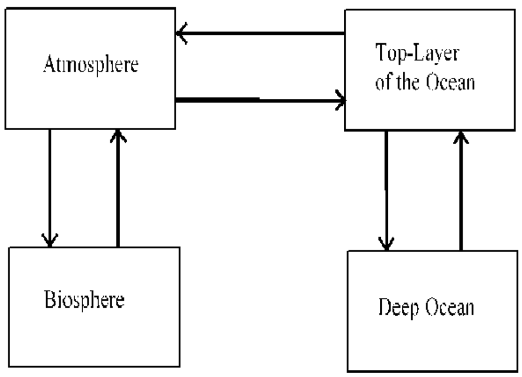
World CO2 Model Simulation

Intro to *Global Carbon-Dioxide (CO2) Model:*

Lumped models are also used to model transactions between different lumped sub-systems. As an example, consider the following diagram which shows the global carbon-dioxide (CO2) cycle on earth where earth is divided into four major compartments, namely, atmosphere, biosphere, the top-layer of the ocean and the deep ocean.



The arrows indicate that there is mass exchange in that direction and the mass exchange between two boxes is given as a rate coefficient ´ current mass. For example, net change in storage in the atmosphere is given as

$$\frac{dx_1}{\mathrm{dt}} = (-k_{12} - k_{13})x_1 + k_{21}x_2 + k_{31}x_3 + 0x_4 \qquad (15)$$

where kij's are rate coefficients for exchange taking place between box i to box j. Carefully note the signs which correspond to the arrows in the figure. The above equation is very similar to other models in this chapter in the sense that it simply equates instantaneous change to net input and the derivatives are with respect to just one indexing variable, here it is time t. Similar equations can be written for the other three compartments giving us the following system of linear differential equations.

$$\frac{\mathrm{dX}}{\mathrm{dt}} = \mathrm{AX} + B \qquad (16)$$

where the matrix A is

$$A = \begin{bmatrix} -k_{12} - k_{13} & k_{21} & k_{31} & 0 \\ k_{12} & -k_{21} & 0 & 0 \\ k_{13} & 0 & -k_{31} - k_{34} & k_{43} \\ 0 & 0 & k_{34} & -k_{43} \end{bmatrix}$$

where X = [x1, x2,x3, x4] are the CO2 stored in atmosphere, biosphere, top and deep layers of the ocean, respectively. The vector B indicates the independence sources corresponding to the four sub-systems. Once we know the rate coefficients and initial conditions we can use this model to estimate CO2 levels in the future.

Example 3.7-Future Predictions of CO2 Levels on Earth: Given that before industrialization (about year 1890), the CO2 levels as = [51 62.2 61.7 2985.4] - giga tons and the rate coefficients as k12 = 1/33, k21 = 1/40, k13= 1/5, k31 = 1/6, k34 = 1/6.2, k43 = 1/300 and that the source term is given as [.05*t 0 0 0] for the first 140 years simulate CO2 levels upto year 2030. The following figure shows results corresponding to the atmosphere. The scaling is in the same order as ppm co2 values in the atmosphere, approximately. So for year 2020, approximately 300-400 (10^10 gigatons) gives a PPM of about 400!

Exercises:

1. Get estimate the initial conditions (more uncertainty there) of this model using observed data
2. What should be the allowed emissions if no more increase in co2 from year 2021?
3. What should be the co2 sequestration rate to reach a ppm of about 350 in 50 years?

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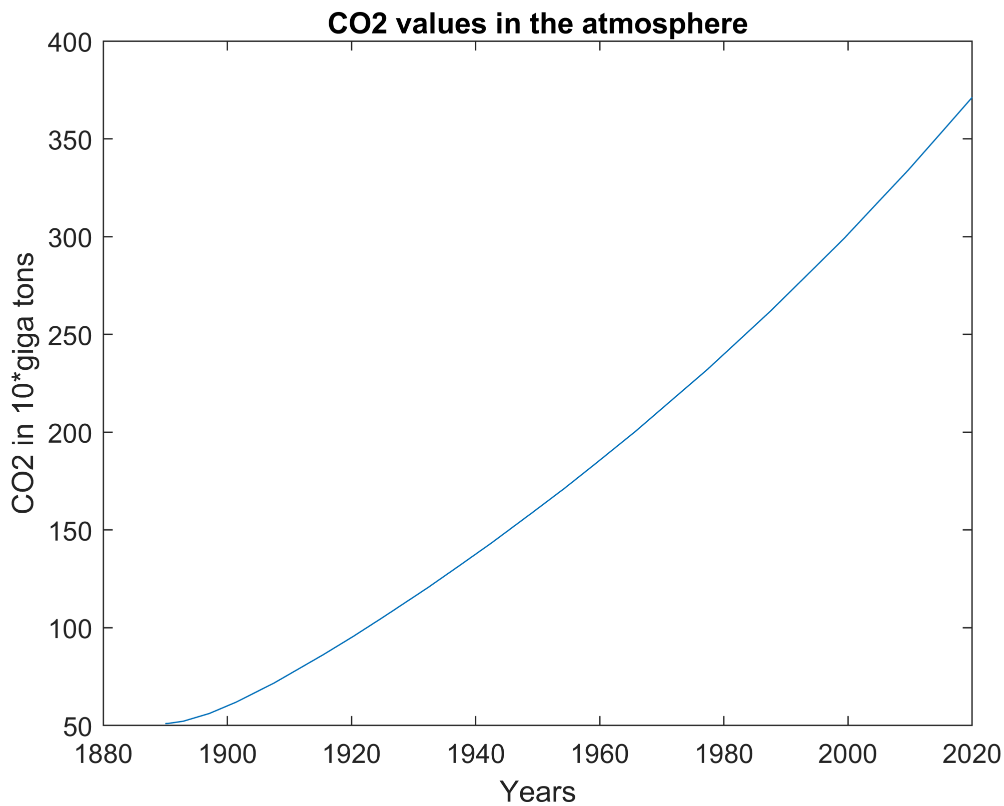
%main program to run the Global CO2
%model

% t0 = 1890;
% tf = 1990;
Years = [1890 2020];
x0 = [51 62.2  61.7 2985.4]'; %CO2 in 10*giga tons

[t,x] = ode23('co2func',Years,x0);

plot(t,x(:,1))


title('CO2 values in the atmosphere')
xlabel('Years')
ylabel('CO2 in 10*giga tons')
```

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%References
%https://www.worldometers.info/world-population/world-population-by-year/
%https://data.worldbank.org/indicator/EN.ATM.CO2E.KT
%Graig H. The natural distribution of radiocarbon and the exchange time of
%carbon dioxide between atmosphere andsea. Tellus. 1957;9:1-17.
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